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(4)

CCS Research Report 584

A NOTE ON THE EFFICIENCY CHARACTERIZATIONS OBTAINED IN DIFFERENT DEA MODELS

by

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June 1988

*Presented at the TIMS/ORSA Conference in Washington, D.C., in April 1988 and at the Conference on Research in Productivity at Rutgers University in December 1987.

The research for this paper was partly supported by National Science Foundation Grant SES 8520806 and U.S. Army Contract DAKF-15-87-C-0110, with The Center for Cybernetic Studies at the University of Texas at Austin. It was also partly supported by the IC^2 Institute at The University of Texas at Austin. Reproduction in whole or in part is permitted for any purpose of the U.S. Government.

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CCS 584

ABSTRACT

Relations of effficiency and non-efficiency for the same sets of DMUs (Decision Making Units) are developed for the CCR, Additive and Multiplicative Models. Surprisingly, additively efficient DMUs are not necessarily multiplicatively efficient. A geometric "stretching" phenomenon is identified for the latter case.

KEY WORDS

Data Envelopment Analysis

CCR, Additive, Multiplicative Models

Efficiency Analysis

1. INTRODUCTION

DEA (Data Envelopment Analysis) first formulated model was a non-linear ratio (or fractional) programming problem, as reported in Charnes, Cooper and Rhodes [14] where it was also shown that the fractional programming transformation first given in Charnes and Cooper [7] could be used to obtain a corresponding dual pair of linear programming problems for use in evaluating the efficiency of not-for-profit entities--such as schools, hospitals and government (including military) agencies. These formulations were to be used to secure efficiency evaluations from observed data on the multiple outputs and multiple inputs generated by the activities of the entities to be studied. Numerical evaluations with operational significance were to be secured without requiring recourse to *a priori* weights (or other transformations) and without requiring explicit specification of parametric functional forms for the relations that might obtain between the inputs and the outputs. Specification was to be required only for the outputs and the inputs to be considered and the DMUs (Decision Making Units) which (1) are responsible for converting inputs into outputs and whose convex or conical combinations (2) constitute the relevant set for obtaining the relative efficiency evaluations that are wanted.

This CCR ratio form was a generalization of the usual single-output-to-single-input ratio definition of efficiency in science and engineering. Embedding this concept in a mathematical programming model (with a corresponding optimization principle), as was done in the CCR ratio model accomplished something more than providing a new way of viewing these time-honored (classical) approaches to efficiency measurement and evaluation. In particular, (1) it provided an opening for extending these ideas for use in situations involving multiple outputs and multiple inputs and (2) it also provided an opening for contacts with other efficiency evaluation concepts such as the Pareto-Koopmans efficiency optimizations in economics, or the vector-optimizations in mathematics.

The CCR ratio form as given in Charnes, Cooper and Rhodes [14] was subsequently extended from the dual side into the BCC form as given in Banker, Charnes and Cooper [4]. This extension was already present in the Charnes, Cooper, Seiford (1981) work (subsequently called the "Additive" model), but the BCC, following the CCR format for getting directly an efficiency measure, introduced a new measuring variable to the linear programming correspond of the CCR formulation. This variable, as

described in Banker, Charnes and Cooper [4] could then be associated with the possible presence of increasing, decreasing and constant returns to scale so that this dimension of efficiency could also be evaluated along with any technical efficiencies (and inefficiencies) that might be present in the activities of any DMU.

The CCR and BCC ratio form development did not exhaust the possibilities for using the underlying DEA concepts and principles.[1] For instance, earlier than the latter form Charnes, Cooper, Seiford and Stutz [12] introduced a multiplicative ratio form and Charnes, Cooper, Seiford and Stutz[13]subsequently extended this to obtain a "units invariant" version of this ratio form. There are various parallels in the way these different ratio formulations utilize the concepts of DEA. Both are invariant and multiple optimizations in both forms--one optimization on each observation--replace the usual once-only optimizations that are common in statistical approaches such those used in least squares regression estimation. In this way, DEA produces efficiency frontier estimates which are piecewise linear for the CCR and BCC models and which are piecewise log-linear for the multiplicative model[2]. In either case these piecewise segments may be regarded as local approximations to the underlying (possibly multiple) functional forms from which the observations were generated.

These topics are studied in depth in Charnes, Cooper, Golany, Seiford and Stutz [11] where it is shown that the basic concepts of DEA are very general and can also be used to provide new approaches and relations to a variety of topics besides efficiency evaluation. Indeed, another "additive model" was introduced in this article which is likewise identified with goal programming and this same formulation and wide generalizations is related to the test for "Pareto efficiency" that was first given in Charnes and Cooper [8] .[3]

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[1] See the basic and earlier introduction of the convex hull by Charnes, Cooper and Seiford (1981) and DEA model identifications with the Charnes-Cooper test for Pareto optimality as extended in Charnes, Cooper, Golany, Seiford and Stutz [11].

[2] See also Banker and Maindiratta [6] for a discussion of how the two can be used together, as may be required when the production frontier is non-concave in some regions.

[3] See also the earlier paper by Charnes and Cooper [9].

2. EFFICIENCY RELATIONS FOR DIFFERENT DEA MODELS

Having identified various forms in which the DEA concepts may be embodied, it is natural to ask about possible relations between them when applied to the same data. This topic is investigated in this paper by reference to the CCR and BCC ratio forms, the additive form, and the multiplicative form. Using $x_{ij} > 0, i = 1, \ldots, m$ for the observed inputs and $y_{ij} > 0, r = 1, \ldots, s,$ for the observed output values for each of $j = 1, \ldots, n$ DMUs we exhibit the DEA models to be investigated in terms of their linear programming equivalents achieved by transformation from their original (ratio and other) forms, as follows:

(1) BCC Ratio Form (modified for units invariance)--Efficiency condition $\theta_o^* = 1$ and all slacks zero.

$$\max -\theta_o + \varepsilon \left[\sum_{r=1}^{s} \frac{s_r^+}{|y_{ro}|} + \sum_{i=1}^{m} \frac{s_i^-}{|x_{io}|} \right]$$

subject to

$$y_{ro} = \sum_{j=1}^{n} y_{rj} \lambda_j \qquad - s_r^+$$

$$0 = -\sum_{j=1}^{n} x_{ij} \lambda_j + \theta_o x_{io} \qquad - s_i^-$$

$$1 = \sum_{j=1}^{n} \lambda_j$$

$$\lambda_j,\ s_r^+,\ s_i^- \geq 0$$

A linear programming equivalent for the CCR Ratio Form is the same as the above with the condition $\Sigma \lambda_j = 1$ omitted.

(2) Additive Form -- Efficiency condition: All slacks equal zero.

$$\max \left[\sum_{r=1}^{s} \frac{s_r^+}{|y_{ro}|} + \sum_{i=1}^{m} \frac{s_i^-}{|x_{io}|} \right]$$

subject to

$$y_{ro} = \sum_{j=1}^{n} y_{rj}\lambda_j \quad - s_r^+$$

$$-x_{io} = -\sum_{j=1}^{n} x_{ij}\lambda_j \quad - s_i^-$$

$$1 = \sum_{j=1}^{n} \lambda_j$$

$$\lambda_j,\ s_r^+,\ s_i^- \geq 0.$$

(3) <u>Logarithmic (Multiplicative) Form</u> -- Efficiency condition: All slacks equal zero:

$$\max \left[\sum_{r=1}^{s} \frac{\hat{s}_r^+}{|\hat{y}_{ro}|} + \sum_{i=1}^{m} \frac{\hat{s}_i^-}{|\hat{x}_{io}|} \right]$$

subject to

$$\hat{y}_{ro} = \sum_{j=1}^{n} \hat{y}_{rj}\lambda_j \quad - \hat{s}_r^+$$

$$-\hat{x}_{io} = -\sum_{j=1}^{n} \hat{x}_{ij}\lambda_j \quad - \hat{s}_i^-$$

$$1 = \sum_{j=1}^{n} \lambda_j$$

$$\lambda_j,\ \hat{s}_r^+,\ \hat{s}_i^- \geq 0,$$

where y_{ro} and x_{io} represent the output and input values for the DMU_o being evaluated. The vertical strokes represent absolute values, which are entered in all three models for consistency in effecting the comparisons that are to be made,[1] and (3) differs from the other models in the above in that it is expressed in logarithmic units, as indicated by the circumflexes over the appropriate variables and constants.

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[1] See Charnes, Cooper, Seiford , and Stutz [13] for a discussion of this device in the additive model to make the resulting efficiency values independent of the units in which the outputs and inputs are measured.

Since the same y_{ro} and x_{io} values appear on the right in each of the above equations we can always obtain a solution by setting the λ_j value associated with these constants at unity. Hence, there is no issue of the existence of solutions in the developments that follow. To be noted also is that $\varepsilon > 0$ is a small, non-Archimedean constant, which is used to accord a preemptive priority status to the choice of θ_o so that the maximization of the objective function in (1) will never cause an increase in the slack values to take precedence over decrease in the value of θ_o.

To start, we have the following:

Theorem 1: DMU_o will be characterized as efficient with the BCC ratio form if and only if it is characterized as efficient with the additive form.

Proof: If in (1), $\theta_o^* < 1$, then

$$\sum_{j=1}^{n} x_{ij}\lambda_j^* \leq \theta_o^* x_{io} < x_{io}$$

Thus λ_j^* together with some positive slacks satisfy the constraints of (2). Thus such an inefficient DMU_o must be rated inefficient by (2), the additive form. Conversely, an efficient DMU_o according to (2) must have $\theta_o^* = 1$. But then with $\theta_o^* = 1$, problems (1) and (2) have the same constraint set and functionals which differ only by a constant. Their optimal solutions $\lambda_j^*, s^{*+}, s^{*-}$ must therefore be the same. Hence DMUs efficient according to (1) are efficient according to (2) and vice-versa.

Q.E.D.

This proof also yields the following:

Corollary 1: DMU_o will be characterized as inefficient by the BCC ratio form if and only if it characterized as inefficient by the additive form.

Corollary 2: The optimal slacks in the additive model will always sum to a value which is at least as great as the sum of the slacks in an optimal solution to the BCC model.

Turning from the BCC to the CCR ratio form, the above theorem is modified to the following:

Theorem 2: If DMU_o is characterized as efficient by the CCR ratio form then it will also be characterized as efficient by the additive model.

Proof: If an optimal basic solution (with $\theta_o^* = 1$, necessarily) does not contain the input-output vector of the efficient DMU_o, then since the "reduced cost" for DMU_o is zero (i.e. its optimal virtual input and output are equal), there is an alternate optimal basic solution (adjacent extreme point) containing this input-output vector. Since an expression of a vector by a basis is unique, $\lambda_o^* = 1$ for this basic optimal solution. Thus there is an equivalent CCR problem for DMU_o with $\sum_j \lambda_j = 1$, hence an identical form to the BCC model. Thus DMU_o is CCR efficient if it is BCC efficient, hence, by Theorem 1, if it is additively efficient.

Q.E.D.

Remark 1: If DMU_o is characterized as inefficient by the CCR ratio form with $\theta_o^* < 1$ and $\sum_{j=1}^{n} \lambda_j^* < 1$, then it will also be characterized as inefficient by the additive form.

The CCR situation with $\theta_o^* < 1$ and $\sum_{j=1}^{n} \lambda_j^* < 1$, the case of locally increasing returns to scale, is more complex and will not be analyzed in detail. The qualitative nature of this relation will depend on the relative magnitudes of $\theta_o^* < 1$ and $\sum_{j=1}^{n} \lambda_j^* < 1$, in a manner that will be apparent when we examine the relations between the logarithmic (multiplicative) and additive models, as we shall do next. We needed to note here the relations between additive and BCC forms because their production possibility sets are different. Cf. the discussion of Figure 1 in Charnes, Cooper, Golany, Seiford and Stutz [11].

To examine the relations between the additive and logarithmic (multiplicative) forms we will need the following lemmas in which we assume that the constants z_k are all positive and the variables w_k satisfy $w_k \geq 0$, $\sum_{k=1}^{t} w_k = 1$. Using "ln" to represent "natural logarithm" we have:

Lemma 1: $\ln\left(\sum_{k=1}^{t} w_k z_k\right) \leq \sum_{k=1}^{t} w_k z_k$

Proof: Since all terms are non-negative, we have

$$\sum_{k=1}^{t} w_k z_k \leq \exp\left(\sum_{k=1}^{t} w_k z_k\right),$$

and therefore

$$\ln\left(\sum_{k=1}^{t} w_k z_k\right) \le \sum_{k=1}^{t} w_k z_k$$

as claimed.

Q.E.D.

Lemma 2: $\sum_{k=1}^{t} w_k \ln z_k \le \ln\left(\sum_{k=1}^{t} w_k z_k\right)$

Proof: Via the geometric and arithmetic mean inequality we have

$$\prod_{k=1}^{t} z_k^{w_k} \equiv z_1^{w_1} z_2^{w_2} \dots z_t^{w_t} \le \sum_{k=1}^{t} w_k z_k$$

with $w_k \ge 0$ all k and $\sum_{k=1}^{t} w_k = 1$. Taking logarithms,

$$\sum_{k=1}^{t} w_k \ln z_k \le \ln\left(\sum_{k=1}^{t} w_k z_k\right)$$

as claimed.

Q.E.D.

Combining Lemmas 1 and 2 then gives

Corollary 2: $\sum_{k=1}^{t} w_k \ln z_k \le \ln\left(\sum_{k=1}^{t} w_k z_k\right) \le \sum_{k=1}^{t} w_k z_k$

Now suppose we have a solution which satisfies the logarithmic model so that for the first $r = 1, \dots, s$ constraints in (3) we have

(4) $$\hat{y}_{ro} \le \sum_{j=1}^{n} \hat{y}_{rj} \lambda_j$$

This same solution will also satisfy the first $r = 1, \dots, s$ constraints in the additive model. This may be shown by arguing from contradiction as follows. Assume that for some r this solution produces

$$y_{ro} > \sum_{j=1}^{n} y_{rj} \lambda_j$$

By virtue of Corollary 2, this gives,

$$\hat{y}_{ro} \equiv \ln y_{ro} > \ln\left(\sum_{j=1}^{n} y_{rj}\lambda_j\right) \geq \sum_{j=1}^{n} \hat{y}_{rj}\lambda_j \tag{5}$$

But since $\ln y_{ro} \equiv \hat{y}_{ro}$ this contradicts the assumption that this solution satisfies (4) for all $r = 1, 2, \ldots, s$ constraints.

This solution for (3) need not satisfy the second set of $i = 1, 2, \ldots, m$ constraints in (2), however, unless it also satisfies the conditions specified in the following:

Theorem 3: Any solution which satisfies the constraints of the logarithmic model will also satisfy the constraints of the additive model if and only if

$$\ln\left(\sum_{j=1}^{n} x_{ij}\lambda_j\right) \leq \ln x_{io}$$

for all $i = 1, 2, \ldots, m$.

Proof: By taking anti-logarithms, it is immediately seen that this choice of λ values satisfies the second set of $i = 1, \ldots, m$ constraints in (2). As was shown by the argument from (4) to (5), it also satisfies the first set of $r = 1, \ldots, s$ constraints with all $\lambda_j \geq 0$ and $\sum_{j=1}^{n}\lambda_j = 1$.

Q.E.D.

To proceed in the other direction we now assume that we have a solution to the additive model so that for the second set of $i = 1, 2, \ldots m$ constraints we have

$$\sum_{i=1}^{m} x_{ij}\lambda_j \leq x_{io} \tag{6}$$

Via Corollary 2 this gives

$$\sum_{j=1}^{n} \hat{x}_{ij}\lambda_j \leq \ln\left(\sum_{j=1}^{n} x_{ij}\lambda_j\right) \leq \ln x_{io} \equiv \hat{x}_{io} \tag{7}$$

so that this solution also satisfies the corresponding constraints in the logarithmic model.

By reasoning as before we then have:

Theorem 4: Any solution which satisfies the constraints of the additive model will also satisfy the constraints of the logarithmic model if and only if

$$\sum_{j=1}^{n} \hat{y}_{rj}\lambda_j \geq \ln y_{ro} = \hat{y}_{ro}$$

Proof: Reason via anti-logarithms as in the proof of Theorem 3.

Q.E.D.

One might be tempted to think that DMU's rated efficient by the CCR form, hence, equivalently via Theorems 1 and 2, by the additive form, should also be efficient going to the multiplicative form since this has been the experience in various real model instances[1]. However, this is not true in general. Witness the following one output, two input example.

	DMU		
	1	2	3
$\hat{y}$	1	1	1
$\hat{x}_1$	2	$2+2\delta^2$	1
$\hat{x}_2$	1	$1-\delta$	2

[1] See, e.g., Ahn [1].

As shown in (3), the constraints are given by

$$
\begin{array}{lllllccc}
 & & & & & & \underline{1} & \underline{2} & \underline{3} \\
1\hat{\lambda}_1 & + & 1\hat{\lambda}_2 & + & 1\hat{\lambda}_3 & -s^+ = & 1 & 1 & 1 \\
-2\hat{\lambda}_1 & - & 2\left(1+\delta^2\right)\hat{\lambda}_2 & - & 1\hat{\lambda}_3 \; -s_1^- & = & -2 & -2\left(1+\delta^2\right) & -1 \\
-1\hat{\lambda}_1 & - & 2\left(1-\delta\right)\hat{\lambda}_2 & & & -s_2^- = & -1 & -1\left(1-\delta\right) & 1 \\
\hat{\lambda}_1 & + & \hat{\lambda}_2 & + & \hat{\lambda}_3 & = & 1 & 1 & 1
\end{array}
$$

with $\quad \hat{\lambda},\ s^+,\ s^- \geq 0$

If, here, $0 < \delta < 1/2$, i.e. $2\delta^2 < \delta$, then DMU_1 is inefficient,

$$\hat{\lambda}_2^* = 1/\left(1+2\delta^2\right), \quad \hat{s}_2^* = \left[(1+\delta)/\left(1+2\delta^2\right)\right]-1, \quad \hat{\lambda}_1^* = \hat{s}_1^{*-} = \hat{s}_1^{*+} = 0 .$$

DMU_2 and DMU_3 are efficient.

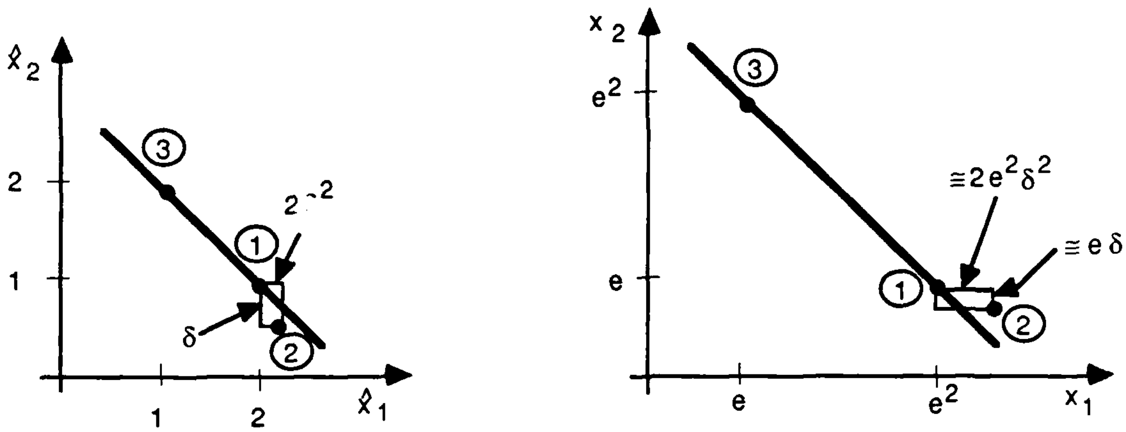
The corresponding additive form (2) has DMU data

	DMU		
	1	2	3
y	e	e	e
x_1	e^2	$e^{2(1+\delta^2)}$	e
x_2	e	$e^{(1-\delta)}$	e^2

All three DMUs are efficient with also (approximately) $2e^2\delta^2 > e\delta$, i.e., $1/2 > \delta > 1/2e$. Thus, DMU_1 is efficient additively but not multiplicatively. The conditions given above for efficiency via the geometric and arithmetic mean inequalities are therefore not vacuous relative to efficiency of DMUs with the additiive and multiplicative forms.

Geometrically this may be seen in the following plot. Since the three DMUs have the same output and therefore also their convex combinations, the envelopment relations may be plotted in the plane of

the two input dimensions. Efficiency means that there are no points in the convex hull to the "southwest" of an efficient point, i.e., that there are no points to the left of the 45° line joining DMUs 1 and 3.



This is false for the multiplicative model and true for the additive model as shown. The "stretching" of scale in going from one to the other is sufficient to reverse the position of DMU 2 relative to the 45° line.

3. SUMMARY AND CONCLUSION

This paper has examined different DEA models with respect to the characterizations of efficiency and inefficiency that may be obtained when these different models are applied to the same data. Other tasks that remain include a study of the differences that might occur in the efficiency values when different forms of DEA all rate a particular DMU as inefficient. Empirical studies, as in Ahn [1], indicate that the values resulting are fairly robust across different models. This is in contrast to the experience with statistical regressions reported in Ahn [1] where different statistical models gave widely varying results (often conflicting even qualitatively) when one class of models was replaced with another to study phenomena such as returns-to-scales, etc. Since there is usually little or no knowledge of the correct functional forms to employ in most studies, especially in studies of not-for-profit entities such as schools, army recruitment, etc., this can be disturbing -- especially when important issues of policy depend on the results of such estimates.

One possibility is to use DEA when the requisite knowledge of parametric forms is not available for use in statistical regressions. Alternatively, the results from a DEA study may be checked with advantage against results from regression studies even when they yield different or conflicting results -- on the principle that " it is better to be confused in the presence of knowledge than to be sure of one's self in its absence." Finally, regression and DEA may be used in combination, as was done by Rhodes and Southwick [18], for example, who used DEA to locate efficiency frontiers and then applied statistical regressions to ascertain whether returns-to-scale possibilities were obtainable from the functional form fitted to these points on the thus identified efficiency frontiers. In such cases, the results in this paper can provide guidance as to when the resulting efficiency characterizations may be expected to be the same and when they might differ from a use of different DEA models.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: CCS 584	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): A Note on the Efficiency Characterizations Obtained in Different DEA Models	5. TYPE OF REPORT & PERIOD COVERED: Technical
	6. PERFORMING ORG. REPORT NUMBER: CCS 584
7. AUTHOR(s): T. Ahn, A. Charnes, and W. W. Cooper	8. CONTRACT OR GRANT NUMBER(s): NSF SES 8520806; DAK-15-87-C-0110
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Center for Cybernetics, UT Austin, Austin, Texas 78712	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: USAREC, Ft. Sheridan, IL 60037	12. REPORT DATE: June 1988
	13. NUMBER OF PAGES: 13
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Data Envelopment Analysis, CCR, Additive, Multiplicative Models, and Efficiency Analysis

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